

On Experimental Evidence of Uncertainty of Quantized Electromagnetic Potential from QHE

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Abstract

It is shown that the observed potential drops on QHE samples can be considered as a realization of uncertainty relation for the quantized two dimensional electromagnetic potential.

Recently it was shown that in accord with the *canonical* flux quantization, there should be an uncertainty relation for quantized two dimensional electromagnetic potential [1]. This uncertainty relation which is comparable with the usual quantum mechanical uncertainty relations is of the form $e\Delta A_x \cdot \Delta x \geq \hbar$, where e , ΔA_x and Δx are, respectively, the electron charge, the uncertainty of electromagnetic potential and the uncertainty of position of electrons. In the context of QHE the position uncertainty of electrons is the width wherein the edge current flows on the *edge* of sample. This width, which is in general larger than the length scale of magnetic length, is given for any QHE sample in accord with the experimental preparation of sample under quantum Hall conditions. In the "ideal" case the edge current flows close to the edge within the length scale of the respective magnetic length l_B [2], which is determined by QHE data of the sample. Its value is defined by: $l_B^2 := \frac{\hbar}{eB}$ or in the single electron picture of IQHE, by $\nu := 2\pi n l_B^2$, where B , ν and n are, respectively, the applied magnetic field, the filling factor and the global density of electrons on the sample.

Here we report on the possibility that the so called potential drops on QHE samples [3] can be considered as realizations of the mentioned uncertainty relation. Recall also that, although electromagnetic potential is not measurable in view of its gauge dependence, nevertheless potential differences are gauge invariant and measurable.

Generally, these experiments can be classified in two groups: One group reports on potential drops which appear on the edge of QHE samples on a width which is equal to the respective $(l_B)^{-1}$ values [3a], [3b]. The other group reports on similar potential drops but on a width which is smaller than the respective $(l_B)^{-1}$ values [3c].

We consider the first type potential drops as the *maximal* and the second type as the *general* potential drops. Thus, we will show that potential drops are (quantum) uncertainty of electromagnetic potential in accord with the mentioned uncertainty relation: Whereby the maximal potential drops appear in cases, where in view of QHE preparation, the position uncertainty of electrons on the sample is the most minimal one which is equal to the magnetic length, i. e. $(\Delta x)_{(minimum)} = l_B$. Whereas the general potential drops are electromagnetic potential uncertainties which appear in cases with a position uncertainty $\Delta x > l_B$.

Therefore, let us briefly sketch the quantum theoretical basis of this uncertainty relation.

The point of departure is the *canonical* conception of flux quantization $\Phi = \oint e A_m dx^m = \int \int e F_{mn} dx^m \wedge dx^n = \mathbf{Z}h$, where A_m and F_{mn} are the electromagnetic potential and the magnetic field strength and $m, n = 1, 2$. We proved that in this case there must be a new uncertainty relation $e\Delta A_m \Delta x_m \geq \hbar$ which is related with the flux quantization $\Phi = \oint e A_m dx^m = \mathbf{Z}h$ and with the quantum commutator postulate $e[\hat{A}_m, \hat{x}_n] = -i\hbar\delta_{mn}$ [1].

To see the consistency of this postulate let us remark that such a commutator can be considered as a result of electronic behaviour in magnetic fields in the following manner:

From the usual requirement in flux quantization that the electronic current density $j_m = ne\hat{V}_m = \Psi^*(\hat{p}_m - e\hat{A}_m)\Psi$ must vanish in the region where the contour integral $\oint A_m dx^m$ takes place, one concludes that *in this region* $[\hat{V}_m, \hat{x}_n] = 0$. This implies that in this region $[\hat{p}_m, \hat{x}_n] = e[\hat{A}_m, \hat{x}_n]$ or that $e[\hat{A}_m, \hat{x}_n] = -i\hbar\delta_{mn}$.

Moreover, it is also known from cyclotron motion of electrons, that the coordinate operators of relative coordinates are non-commuting [4a]. Thus, one has $[\hat{x}_m, \hat{x}_n] = -il_B^2\epsilon_{mn}$ for the relative cyclotron coordinates, where l_B is the magnetic length [4b]. Now this commutator is proportional to the mentioned commutator $[\hat{A}_m, \hat{x}_n] = -i\delta_{mn}\hbar$ by the usual Landau gauge $A_m = Bx^n\epsilon_{mn}$, $\epsilon_{mn} = -\epsilon_{nm} = 1$.

We showed also rigorously that the flux quantization can be understood as the canonical quantization of flux functional $\Phi = \oint e A_m dx^m = \int \int e F_{mn} dx^m \wedge dx^n$ on the phase space of flux system which contains the set of canonical conjugate variables $\{A_m, x^m\}$. Thus, in accord with geometric quantization [5] the quantum differential operators on the quantized phase space of this system should be given by $\hat{A}_m = -i\hbar\frac{\partial}{\partial x^m}$ and $\hat{x}_m = i\hbar\frac{\partial}{\partial A_m}$ [1]. Recall however that the wave function of quantized $\{A_m, x^m\}$ system should be considered either in the $\Psi(A_m, t)$ - or in the $\Psi(x^m, t)$ representation. Therefore, the quantum operators should be given, respectively, either by the set $\{\hat{A}_m = A_m, \hat{x}_m = i\hbar\frac{\partial}{\partial A_m}\}$ or by the set $\{\hat{A}_m = -i\hbar\frac{\partial}{\partial x^m}, \hat{x}_m = x^m\}$ [1].

In both representations the commutator between the quantum operators is given by $(-i\hbar)$:

$$e[\hat{A}_m, \hat{x}_n]\Psi = -i\hbar\delta_{mn}\Psi \quad (1)$$

Equivalently, we have in accord with quantum mechanics a true uncertainty relation for A_m and x_m , i. e.: $e\Delta A_m \cdot \Delta x_m \geq \hbar$.

Furthermore, the electromagnetic gauge potential have in accord with the uncertainty relations $e\Delta A_m \cdot \Delta x_m \geq \hbar$ a maximal uncertainty of $(\Delta A_m)_{(maximum)} = \frac{\hbar}{el_B}$ for the case where the position uncertainty aquires its most minimal value. This "ideal" case where $(\Delta x_m)_{(minimum)} = l_B$ corresponds to the "uncertainty equations" $e\Delta A_m \cdot \Delta x_m = eB\Delta x_m \cdot \Delta x_n |\epsilon_{mn}| = \hbar$, from which one can obtain the independent definition of magnetic length $l_B^2 = \frac{\hbar}{eB}$ [1]. This procedure proves the consistency of the approach.

Hence we show that the observed potential drops in QHE experiments which is reported in [3] can be considered as experimental evidences for the above uncertainty relation.

From topological point of view, which is useful in a topological effect like QHE, all usual two dimensional QHE samples are equivalent to a disc. Thus, one should consider QHE on such a sample with the radial (r) and azimuthal (ϕ) degrees of freedom, where $(\{x_m\} \sim \{r, \phi\})$ and $(\{A_m\} \sim \{A_r, A_\phi\})$.

The uncertainty relation $e\Delta A_m \cdot \Delta x_m \geq \hbar$ asserts that the general potential uncertainty is given by $\Delta A_m \geq \frac{\hbar}{e\Delta x_m}$. If we identify the azimuthal or the *edge* component A_ϕ with Hall potential A_H , then the uncertainty relation for the Hall potential is given by $\Delta A_H \cdot \Delta x_H \geq \frac{\hbar}{e}$ where Δx_H is the position uncertainty for electrons which flow in Hall current on the edge of sample. Thus, the general potential uncertainty for Hall potential is given in accord with the uncertainty relation by $\Delta A_H \geq \frac{\hbar}{e\Delta x_H}$, whereas the maximal potential uncertainty for A_H is given by $(\Delta A_H)_{(maximum)} = \frac{\hbar}{el_B}$ for the case $(\Delta x_H)_{(minimum)} = l_B$.

Now, in the first group, Ref. [3b] reports on observation of potential drops across the QHE samples over a width of $100 \mu m$ from the edge of samples. This experiment is performed on QHE samples with filling factor $\nu = 2$ and a magnetic length value of $l_B = 10^{-2} \mu m$ [3b].

On the other hand, the theoretical value for the maximal potential uncertainty for Hall potential which is obtained from uncertainty relation is $(\Delta A_H)_{(maximum)} = \frac{\hbar}{el_B}$. Therefore, one obtains with $l_B = 10^{-2} \mu m$ the value $(\Delta A_H)_{(maximum)} \approx 100 \mu m$ [6].

It agrees with the observed width of potential drop in Ref. [3b].

The other experiment in the first group [3a] is performed under almost the same QHE conditions as in Ref. [3b], but with filling factor $\nu = 4$. The value of magnetic length for this sample is obtained to be $l_B \approx 1.4 \cdot 10^{-2} \mu m$ in accord with QHE data in Ref. [3a]. Therefore, we obtain for the theoretical value of $(\Delta A_H)_{(maximum)} = \frac{\hbar}{el_B}$ in this case $(\Delta A_H)_{(maximum)} \approx 70 \mu m$ [6]. It agrees also with the observed value in Ref. [3a].

In this sense, from theoretical point of view, our maximal Hall potential uncertainty results which correspond to the respective most minimal position uncertainties or to the respective magnetic lengths, agree with the observed results of potential drops in experiments of the first group [3a], [3b].

Moreover, since the ratio between the calculated value of magnetic lengths from experimental data of the first group $\frac{(l_B)_{[3a]}}{(l_B)_{[3b]}} \approx 1.4$ is equal to the ratio between their respective filling factors $(\frac{\nu_{[3a]}}{\nu_{[3b]}})^{\frac{1}{2}} = (\frac{4}{2})^{\frac{1}{2}} \approx 1.4$. Then it is important to mention that in this group, where the samples differ almost only with respect to the actual filling factors, the ratio between the width of the observed potential drops $\frac{(\Delta A_H)_{(maximum)[3b]}}{(\Delta A_H)_{(maximum)[3a]}} \approx 1.4$ is equal to the reciprocal ratio of the respective filling factors and the respective magnetic lengths, i. e.:

$$\frac{(\Delta A_H)_{(maximum)[3b]}}{(\Delta A_H)_{(maximum)[3a]}} \approx (\frac{\nu_{[3a]}}{\nu_{[3b]}})^{\frac{1}{2}} \approx \frac{(l_B)_{[3a]}}{(l_B)_{[3b]}}. \quad (2)$$

Thus, not only that the observed ratio between potential drops in this group agrees with the theoretical ratio between respective potential uncertainties, but this agreement proves even the exclusive reciprocal dependence of *maximal* potential drops from the respective filling factor or equivalently from the respective magnetic length, if as in the experiments of this group other relevant data are almost the same.

Nevertheless, it is possible that under QHE conditions the electronic edge current flows, not within the length scale of magnetic length, but further within a larger length scale on the sample. Then the position uncertainty of electrons which is the actual length scale wherein the edge current flow, have its general value which is larger than that of the magnetic length of sample $\Delta x_H > l_B$. Therefore, the value of uncertainty of Hall potential is in this cases less than its maximum value, i. e. $(\Delta A_H)_{(general)} <$

$(\Delta A_H)_{(maximum)} = \frac{\hbar}{el_B}$. In other words in accord with the uncertainty relation potential uncertainties or potential drops on QHE samples should be observed on a width which are, in general, smaller than the respective $\frac{\hbar}{el_B}$ values. This is what is varified in some of the experiments of second group in Ref. [3c].

Thus, our theoretical results for the general potential uncertainty also agree with the observed results of the second group in Ref. [3c] where they report on potential drops on QHE samples, which is less than the respective $\frac{\hbar}{el_B}$ values.

Footnotes and references

References

- [1] F. Ghaboussi, quant-ph/9702054; cond-math/9703080.
- [2] K. v. Klitzing, Physica B 204, 111-116 (1995).
- [3] , [3 a] P. F. Fontein, et al., Phys. Rev. B., 43, 12090 (1991).

The relevant QHE data in this report for our calculation are $n = 5.0 \cdot 10^{15} m^{-2}$ and $\nu = 4$. This corresponds, in accord to $\nu = 2\pi n l_B^2$, to a magnetic length value $l_B \approx 1.4 \cdot 10^{-2} \mu m$.

[3 b] W. Dietsche, K. v. Klitzing and K. Ploog, Surf. Sci. 361/362, 289-292 (1996).

The relevant QHE data in this report for our calculation are $n = 3.7 \cdot 10^{11} cm^{-2}$ and $\nu = 2$. This corresponds to a magnetic length value $l_B \approx 10^{-2} \mu m$.

[3 c] Further reports on potential drops are:

G. Ebert, K. v. Klitzing and G. Weimann, J. Phys. C.: Solid state Phys., 18 (1985) L257-L260;

H. Z. Zheng, et al. Phys. Rev. B 32 (1985) 5506;

E. K. Sichel, et al. Phys. Rev. B 32 (1985) 6975;

P. F. Fontein, et al. Surface Scince 263 (1992) 91;

S. Takaoka, et al., Phys. Rev. Lett 72 (1994) 3080.

[4] , [4a] Landau-Lifschitz, "Quantum Mechanics (Non-Relativistic Theory)" Vol.III (Pergamon Press 1987).

[4b] H. Aoki: Rep. Prog. Phys. 50, (1987), 655.

[5] N. Woodhouse,"Geometric Quantization", (Clarendon Press, 1980, 1990) Oxford University.

[6] Recall that in geometric units, where $(mass \sim L^{-1})$, $(length \sim time \sim L)$ and $(e \sim L^0)$, the action \hbar is of dimension L^2 . Hence, $\Delta A \sim \frac{\hbar}{el_B}$ is of dimension L , which agrees with dimension of the observed width of potential drops. Recall also that, in view of dynamical behaviour of electrons in a magnetic field, one should consider the electro-dynamical value $(e \approx 1.6 \cdot 10^{-19} \text{ Amper} \cdot S)$ in $\frac{\hbar}{e}$. Then in accord with $\hbar \approx 1.6 \cdot 10^{-27}$ one has in the (C. G. S.) system $\frac{\hbar}{e} \approx 10^{-8} \text{ cm}^2 = 1 \text{ } \mu m^2$.